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A mixed-integer linear programming model for grain shipment in black sea grain

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Abstract

The Black Sea Grain Initiative is proposed by the United Nations in light of the Russo-Ukraine War, which has disturbed the transportation of grains from Russia to the global market. This paper examines the shipments under this program, which involves the delivery of various types of grains to multiple destinations over an extended period. A mixed-integer linear programming model is formulated to determine the minimum total cost, which consists of both shipment costs and demand-based compensation, considering several factors including the supply, demand, vessel speed and capacity, and the income level of destination countries. When solved, the model finds the type of grain transported, the destination, the vessel deployed, and the starting date of each voyage. A sample problem is generated and solved to demonstrate the feasibility of the model. Numerical results and sensitivity analysis is then presented to further support the significance of the cost components and the validity of the approach.

Keywords: Black Sea Grain Initiative; Vessel scheduling; Maritime transportation; Mixed-integer linear programing

Jel codes: A12, C61, C51

1. Introduction

This inspiration of the paper stems from the Black Sea Grain Initiative, which was proposed by the United Nations in 2022 in response to the Russo-Ukraine War, which has caused great disturbance to the transportation of essential commodities, including food and fertilizer, from Russia to the global market. As of April 2023, the program has enabled shipment of over nearly 27 million tons (UN, 2023). Considering the high operating costs of vessels, which amounts up to more than a hundred thousand dollars daily (Ronen, 2011), it is crucial to schedule shipments effectively. Moreover, as the Black Sea Grain Initiative facilitates the delivery of grains as aid to countries with food insecurity, proper shipment planning also holds humanitarian significance.

The shipping network comprises a loading port, a number of destinations, and vessels sailing round trips between the port and a destination. In the case of the Black Sea Grain Initiative, there are three loading ports, namely Odesa, Chornomorsk, and Yuzhny/Pivdennyi (UN 2022). Since the distance between these ports is negligible compared to their distance from destinations, the three ports are considered as a single port.

The process of arranging shipments takes into consideration several factors, including the supply of different types of grains at the port, vessel availability, and demand satisfaction over a specified period. The problem is solved to achieve minimization of the total cost, which is the sum of shipment costs and demand-based compensation, which consists of both unmet demand penalty and exceeded demand award.

This paper formulates a mixed-integer linear programing (MILP) model to address the problem, which can be applied not only to the Black Sea Grain Initiative, but also other liner shipping network design problems with suitable modifications. This paper also highlights the distinctive features of the proposed optimization model for solving a vessel arrangement problem of both business and humanitarian causes.

The paper is organized as follows. Section 2 provides a review of relevant literature. Section 3 describes the problem and outlines certain assumptions and delimitations. The mathematical model is formulated in Section 4, and a small example problem is solved in Section 5. Lastly, Section 6 gives some closing comments.

2. Literature Review

Maritime transportation is an important aspect of international trade, as it is responsible for around 90 per cent of global trade (Bocanegra, 2010). In recent years, there has also been a growing interest associated with maritime transportation optimization (Christiansen, Fagerholt, Ronen, 2004). One particular area of study focuses on specific route design on a weekly basis, also known as liner shipment network design problems (Christiansen, Hellsten, Pisinger, Sacramento, Vilhelmsen, 2020). Many researchers including Brouer, Desaulniers, Pisinger (2014), Reinhardt, Kallehauge, and Pisinger (2010), have contributed to this field. However, the focus of this paper is rather on shipment scheduling with pre-determined routes. Research on vessel planning can be traced back to World War II, when logistics problems necessitated optimization methods (Dantzig (1914-2005)). Various optimization techniques have since been applied in related research. Some examples are stochastic optimization (Adriana, Bervery, Hugo, 2013) which introduced uncertainty and dynamic programing (Anily, Tzur, 2005). Among these, linear programing has also been widely used by scientists in this area. For instance, Charnes and Cooper (1954) discussed about the application of linear programing in shipment scheduling, while Cundiff, Dias, and Sherali (1997) presented a model of linear programing to solve a biomass delivery design problem at a plant in Virginia. LeBlanc and Hill (2004) developed spreadsheet linear programing models to investigate optimization of the supply chain cost for the firm Nu-kote International. Bilgen and Ozkarahan (2007) used mixed-integer linear programing to study the supply chain of wheat of a company, including both blending and shipping.

Since the Black Sea Grain Initiative is a recent program, there are few papers dedicated to investigate maritime transportation involved. This paper aims to provide an insight into shipments in this specific initiative.

In problems related to transportation planning, it can be extremely difficult to meet the exact demand. Thus, an unmet demand penalty is often incorporated when forming the model, as an additional cost incurred, which can be found in papers of Brown, Keegan, Vigus, and Wood (2001) and Adriana, Paulo, Bervery, Irineu, Hugo (2021). The model in this paper differs from others as it includes an award for exceeded demand. Since the Black Sea Grain Initiative also facilitates grain transportation for charity other than business, the award especially helps to encourage delivery of food to countries with hunger issues.

3. Problem statement

In this paper, a mixed-integer linear programming model of liner shipping network design problem with a focus on vessel allocation and an object of cost minimization is studied. This model is concerned with the shipment planning of multiple types of grains from a loading port to destinations over a specific time frame. The cost comprises the shipment cost and demand-based compensation, which consists of both penalty for unmet demand and award for exceeded demand. Vessel availability as well as the demand and supply of different grains at a port and destination in a period is based on the real-life Black Sea Grain Initiative with slight modifications.

To provide additional clarity on the issue at hand, the following considerations provide further detail and context:

- A voyage is defined by the type of grains carried, the destination, the vessel, and the start date.
- Multiple identical vessels with the same velocity and capacity are on a shipping voyage together at a time. On the voyage, the vessels only deliver one type of grain to one destination, starting on the same day.
- The shipping route is fixed with the port and a destination.
- Day 1 in the model is 1 August 2022 in real life, as the Black Sea Grain Initiative commenced in August 2022.
- The time unit related to voyage is in days, while the time unit related to demand and supply is in months.
- The range of the demand and supply is determined from the amount shipped from a port or to a destination in the actual initiative.
- All vessels are assumed to be at the port on day 1. They go to the destination and immediately returns to the original port. The time taken for loading is not considered separately. They are on a constant speed, without stopping during a voyage.
- The departure and return voyages take the same number of days, which is determined by the voyage distance and vessel speed.
- The vessels always carry grains to their full capacity.
- The result is not directly in dollars or any other currencies. However, to show its monetary nature, $ is used for any price-related variable.

Since multiple identical vessels are on a voyage simultaneously, in the model formulation the capacity parameters, including the amount shipped from the port and shipped to a destination on a voyage is multiplied by the number of vessels. The speed of vessels deployed affects the shipment cost, while their capacity determines the penalty or award for the gap between amount shipped in the model and the demand. Therefore, the variety of vessels is a key component in modelling and has a significant impact on the optimal solution.

Figure 1. presents the input and output of the model.

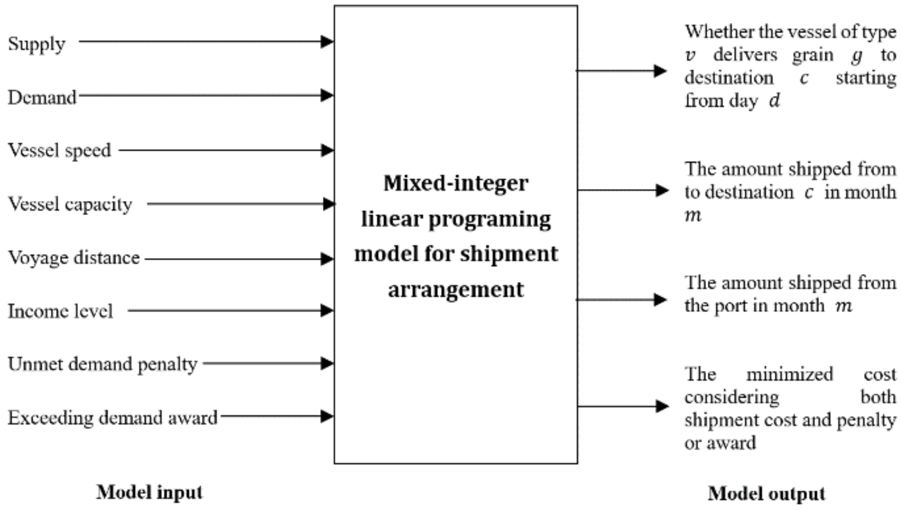


Figure 1. Model input and output

4. Mathematical model

This section presents the mixed-integer linear programming model for shipment arrangement. It includes the description and explanation of the notations, variables, constraints, and the objective.

4.1. Notation and definitions of parameters and decision variables

The parameters and variables both have multiple indices since the model involves both places and time. Table 1 presents a list of notations.

Table 1. Notations

Notation	Remark
Index sets	
G	Set of types of grains $\{1, 2, \ldots, G\}$
C	Set of destinations $\{1, 2, \ldots, C\}$
V	Set of vessels $\{1, 2, \ldots, V\}$
D	Set of days $\{1, 2, \ldots, D\}$
M	Set of months $\{1, 2, \ldots, M\}$
I	Set of income levels $\{1, 2, \ldots, I\}$
Input parameters	
De_{gcm}	Demand for grain g by destination c in month m in tonnage
Su_{gm}	Supply of grain g in month m in tonnage
VS_v	Speed of vessel v in nautical mile per hour
$VCap_v$	Capacity of vessel v in tonnage
$CDis_c$	Distance from the port to destination c in nautical mile
CIn_c	Income level of destination c
Pe_i	Unmet demand penalty per tonnage for destinations of income level i
Aw_i	Exceeded demand award per tonnage for destinations of income level i
F_m	The first day of month m
L_m	The last day of month m
k	The number of identical vessels on a voyage simultaneously
Decision variables	
x_{gcvd}	Binary variables of whether the vessel of type v delivers grain g to destination c starting from day d
t_{gcm}	Amount of grain g shipped to destination c in month m
f_{gm}	Amount of grain g shipped from the port in month m
h_{gcm}	Demand-based compensation of grain g in destination c in month m
Logical operators	
$[Expn]$	Judged whether expression $Expn$ is true or false, represented by 1 or 0 respectively

Shipment planning is implemented through the binary variable x_{gcvd}, which indicates whether there exists a voyage of vessel v delivering grain g to destination c starting from day d. The subscripts help identify relative data which is used in calculation in constraints and the objective function. The model can thus determine the voyage distance, the voyage duration, and the amount shipped.

t_{gcm} and f_{gm} keeps track of the amount of a type of grain shipped to a destination and from the port respectively. They can be calculated by x_{gcvd} and $VCapacity_v$, as all vessels are assumed to carry grains to its full shipload.

h_{gcm} helps determine the demand-based compensation. When the amount shipped does not meet the demand, a penalty is imposed as a positive compensation, while when the amount shipped exceeds the demand, an award is given as a negative compensation. Considering their nature, the penalty and award are in different numerical values. Thus, the demand-based compensation would not be a linear function. To ensure the mixed-integer linear programing model can be used, the slack variable h_{gcm} is introduced, which will be further explained in 4.3. Constraints.

4.2. Verbal model

A verbal description of the linear programing model is presented as follows:

Minimize:

Total cost = Shipment cost + Demand-based compensation, Subject to:

Vessel availability constraints

Demand constraints

Supply constraints

The problem discussed in this paper is conceptually simple, but mathematically challenging. Thus, the mathematical model formula is especially important, and will be discussed in detail in the following two sectors.

4.3. Expressions

Variables t_{gcm}, f_{gm}, and h_{gcm} are to be calculated from other decision variables and input parameters.

4.3.1. f_{gm}

As f_{gm} is the amount of grain g shipped from the port in month m, it is expressed as the sum of all vessel capacities, if the vessels are on a voyage delivering grain g and the starting date d is in month m. Thus, f_{gm} is expressed as

$$f_{gm} = k \sum_{v=1}^{V} \sum_{c=1}^{C} \sum_{d=F_m}^{L_m} \left(x_{gcvd} VCap_v \right)$$

4.3.2. t_{gcm}

Similarly, as t_{gcm} is the amount of grain g shipped to destination c in month m, it is the sum of all vessel capacities, if that vessel is delivering grain g to destination c and the date of arrival is in month m. However, since arrival in month m does not necessarily mean departure in month m, there is an additional part compared to the expression for f_{gm}, which checks whether the date of arrival is in month m. Thus, the expression is

$$t_{gcm} = k \sum_{v=1}^{V} \sum_{d=1}^{L_m} \left(x_{gcvd} VCap_v \left[F_m \leq d + \left\lceil \frac{CDis_c}{24VS_v} \right\rceil \leq L_m \right] \right)$$

The expression $\left\lceil \frac{CDis_c}{24VS_v} \right\rceil$ is the duration of the voyage in days. It always rounds up to compensate for the time of loading and unloading.

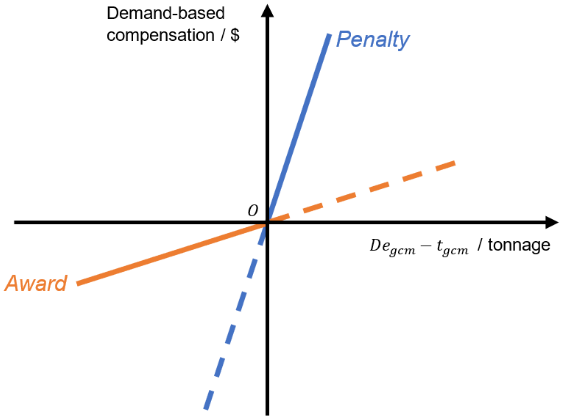
4.3.3. h_{gcm}

h_{gcm} is a slack variable to take into account for demand-based compensation, which can be in form of award or penalty. The gap between the amount shipped to a destination and the demand is $De_{gcm} - t_{gcm}$. When this value is positive, meaning that the demand is not met, a penalty is imposed as an addition on the total cost. Conversely, when the value is negative, meaning that the amount shipped exceeds the demand, an award is given as a deduction from the cost. This can be represented in Figure 2.

As shown in Figure 2, the demand-based compensation, represented by the solid lines, is a piecewise function. Aw and Pe are set at different values according to the income level of destination countries. Meanwhile, for a specific income level category, Pe should always be larger than Aw, so as to make the amount shipped as close to the demand as possible but preferably higher. The difference in gradients leads to the convex shape of the function, as the demand-based compensation is always the larger value of penalty and award, regardless of being penalty or award. Thus,

$$h_{gcm} = \max\left(Pe_{CIn_c}(De_{gcm} - t_{gcm}), Aw_{CIn_c}(De_{gcm} - t_{gcm})\right)$$

Figure 2. Demand-based compensation



To make the expression linear, the following constraints are proposed:

$$\begin{cases} h_{gcm} \geq Pe_{CIn_c}(De_{gcm} - t_{gcm}) \\ h_{gcm} \geq Aw_{CIn_c}(De_{gcm} - t_{gcm}) \end{cases}$$

4.4. Constraints

Constraints are applied to ensure that the results of the model are applicable and also in line with the actual Black Sea Grain Initiative.

4.4.1. Vessel availability constraint

This consideration is rather straightforward: for a vessel v on day d, it can only be on one voyage at most. The total number of voyages of a vessel v on day d is given by

$$\sum_{g=1}^{G}\sum_{c=1}^{C}\sum_{s=1}^{d} x_{gcvs}\left[s \leq d \leq s + 2\left\lceil\frac{CDis_c}{24VS_v}\right\rceil\right]$$

The iteration of s, which represents the starting day of a voyage, is from day 1 to the current day d. When the vessel v starts its voyage on day s and day d is in between the start and return day of the voyage, vessel v is on one more voyage on day d. Thus, the constraint on vessel availability is written as

$$\sum_{g=1}^{G}\sum_{c=1}^{C}\sum_{s=1}^{d} x_{gcvs}\left[s \leq d \leq s + 2\left\lceil\frac{CDis_c}{24VS_v}\right\rceil\right] \leq 1$$

4.4.2. Supply constraint

It is set that the amount of grain g shipped out in month m should not exceed 150% of the supply of grain g in month m. The amount shipped out should not be larger than the original supply by a great amount. However, there should also be some tolerance on the range, as it would not impose great difficulty to increase the supply of grains to the port by some extent. Thus, a range of 150% is allowed by the amount shipped from the port in the model, expressed as

$$f_{gm} \leq 1.5Su_{gm}$$

4.4.3. Demand constraint

Likewise, some fluctuation should also be granted to the demand. The amount of grain g shipped to destination c in month m should at least be 70% of the original demand. However, it should also not exceed the demand by too much, which leads to overstock. Moreover, the exceeded demand award can be exploited to minimize the total cost to unreasonably low if there is not an upper limit. Thus, a range of 70%~130% is set to the amount shipped to a destination, expressed as

$$0.7De_{gcm} \leq t_{gcm} \leq 1.3De_{gcm}$$

4.5. Objective function

The objective of the model is to minimize the total cost, which consists of both shipment costs and demand-based compensation. It is generally assumed that the bunker consumption per day is linearly related to third power of the vessel velocity (Dulebenets, 2015). Furthermore, according to most estimations, the bunker cost comprises approximately 50-60% of total operating cost. Since the model outputs the cost in unit price instead of an accurate value in dollars, the coefficients can be omitted. Hence, the objective function writes

$$\sum_{g=1}^{G}\sum_{c=1}^{C}\sum_{v=1}^{V}\sum_{d=1}^{D} x_{gcvd}\left\lceil\frac{CDis_c}{24VS_v}\right\rceil {VS_v}^3 + \sum_{g=1}^{G}\sum_{c=1}^{C}\sum_{m=1}^{M} h_{gcm}$$

5. Computational study and results

5.1. Input parameters

To better illustrate how the model can be applied, a representative problem is constructed. Considering the high dimensions of the decision variables and the potential problem matrix, the sets are made small compared to the real-life scenario. The sizes of the sets are listed in Table 2.

Table 2. Set sizes

Set	Size
G	3
C	10
V	157
D	61
M	2
I	4

As this model investigates shipment in the Black Sea Grain Initiative, the input parameters are mostly based on the official data from the United Nations and processed. The subscripts g and c are according to alphabetical order of grains and destination countries respectively, and v is according to ascending order of the vessels' IMO number. While VS_v, $VCap_v$, $CDis_c$, F_m, and L_m can be directly looked up from online on websites including http://ports.com/sea-route/ and https://www.hifleet.com/, De_{gcm} and Su_{gm} can be calculated from the shipment data in the initiative provided by United Nations. k is set to be 3. Other inputs that require further procession are listed in Table 3.

Table 3. Input parameter values

Parameter	Value	Remark
CIn_c	$CIn_c \in \{1, 2, 3, 4\}$	$\{1, 2, 3, 4\}$ corresponds to {low income, lower-middle income, upper-middle income, high income} respectively.
Pe_i	$[80, 60, 40, 20]$	The highest bunker cost per tonnage possible is $\frac{\left\lceil\frac{CDis_{max}}{24VS_{max}}\right\rceil VS_{max}{}^3}{VCap_{min}}$, which equals 18.8. Penalties should always be higher than this value, and penalties for countries with a lower income level should be higher.
Aw_i	$[0.012, 0.09, 0.06, 0.03]$	The lowest bunker cost per tonnage possible is $\frac{\left\lceil\frac{CDis_{min}}{24VS_{min}}\right\rceil VS_{min}{}^3}{VCap_{max}}$, which is 0.0124. Awards are smaller than this value, and awards for countries with a lower income level should be higher.

5.2. Computational results

The mixed-integer linear programming model discussed above was implemented using Programming language Julia 1.8.3 (2022-11-14), and the problem was solved using a modeling language for mathematical optimization, namely JuMP 1.4.0 (2022-10-29). The computational tests are performed on a Core i5 notebook with a 1.6GHz processor and 8 GB of RAM.

The model minimizes the cost to be 254229.82 units price with 262003.00 shipment costs and -7773.18 demand-based compensation. The vessel arrangement is presented in table 4.

Table 4. Vessel arrangement ($x_{gcvd} = 1$)

v (Vessel No.)	d (Starting day)	g (Grain type)	c (Destination)
9	34	1	6
12	1	2	10
53	1	2	7
57	3	1	6
58	48	2	10
59	31	1	6
63	36	2	7
79	31	1	6
89	31	2	6

5.3. Impact of demand-based compensation

To gain an insight into the effect of demand-based compensation on the result, the model is modified. Firstly, the compensation h_{gcm} is excluded, making only shipment cost considered. In this case, the result is 230529 units price. The vessel arrangement is listed in table 5.

Table 5. Vessel arrangement ($x_{gcvd} = 1$) with no demand-based compensatio

v (Vessel No.)	d (Starting day)	g (Grain type)	c (Destination)
12	1	2	10
45	48	2	10
57	3	1	6
59	31	1	6
63	1	2	7
89	35	2	6
92	31	2	7

Next, while demand-based compensation is included again, only the penalty is imposed, while there is no award for exceeding the demand. This is implemented by replacing the constraint

$$h_{gcm} \geq Aw_{CIn_c}(De_{gcm} - t_{gcm})$$

with

$$h_{gcm} \geq 0$$

The result of the modified model is 261539. The new vessel arrangement is listed in table 6.

Both the difference in minimized cost and vessel arrangement demonstrates the significance of demand-based compensation. It makes the solution more practical and serves for humanitarian good, since the compensation depends on the income level of destination countries.

Table 6. Vessel arrangement ($x_{gcvd} = 1$) with only unmet demand penalty

v (Vessel No.)	d (Starting day)	g (Grain type)	c (Destination)
12	34	2	6
58	10	2	10
59	1	1	6
61	30	1	6
79	21	2	10
85	1	2	7
85	36	2	7
99	48	2	10

5.4. Sensitivity analysis

A sensitivity analysis provides an insight into how input parameters, namely shipment costs and demand-based compensation in this case, affects the result of the model. Since the solver HiGHS does not support sensitivity analysis of discrete variables, the sensitivity report is generated manually by increasing shipment cost and demand-based compensation by increments of 5% respectively. The result is presented in table 7.

Table 7. Change in total cost with increase in cost components

Percentage increase (%)	Shipment cost		Demand-based compensation	
	New total cost	Change in total cost (%)	New total cost	Change in total cost (%)
5	267329.97	5.2	253841.2	0.153
10	280430.12	10.3	253452.5	0.306
15	293530.27	15.5	253063.8	0.459
20	306630.42	20.6	252675.2	0.612
25	319730.57	25.8	252286.5	0.764

As demonstrated in table 7, the total cost is more susceptible to change in shipment cost. A 5% increase in shipment cost causes the total cost to rise by 5.2%, while a 5% increase in demand-based compensation only causes the total cost to change by 0.153%. Other values of increments also follow this trend. Thus, the result generated by the model is reasonable, as the shipment cost has a larger impact on the total cost, while the demand-based compensation encourages exceeding demand within the limit without drastically altering the total cost.

6. Conclusion

This paper develops a real vessel scheduling model to determine what vessels are deployed, types of grains to be transported, the destination, and the starting date of voyages so as to minimize the total cost while meeting most demands.

This paper has two main distinctive features. First, it integrates demand-based compensation, which includes both penalty for unmet demand and award for exceeded demand. It helps to prioritize demand of countries of lower income level, which have a higher chance of facing acute food insecurity. Second, it focuses on the Black Sea Grain Initiative, which is a new program without many relevant research and also significant in current affairs.

An extension of the model is to consider multiple ports. This change will enable the model to be applied to transportation systems with ports that scatter around the globe. Another extension is to allow a vessel to carry more than one type of grains. This can help to reduce the number of vessels shipping to the same destination in a relatively short time frame by deploying vessels of larger capacities.

In conclusion, the model proposed in this paper can be an effective tool in vessel scheduling in the Black Sea Grain Initiative. With modifications, it can also be applied to other planning problems in maritime transportation.

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